Deformable modeling

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Overview

- Motivation
  - Why deformable models
- Types of deformable models
  - Most commonly encountered
  - Variants
  - Examples in the literature
- Key research issues

Why deformable models?

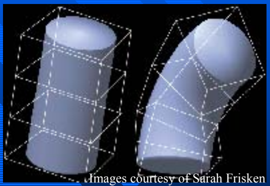
- Human tissues are soft
- Need to model their appearance and mechanical properties in a realistic fashion
- Need to compute reaction forces for haptic feedback

An ideal deformable model

- Fast
  - » Visual update rate: >= 10Hz
  - » Haptic update rate: >= 1000Hz (approximation/interpolation schemes can reduce this somewhat)
- Realistic
  - » Tissues are inhomogeneous
  - » Interaction with other organs, tools
- Facilitate cutting and suturing

Types of deformable models

- Non-physics based models
  - Splines, patches, snakes, free-form deformation
  - Parameters required to deform model may not be intuitive
  - May be sufficient for some simulations
  - Example
    - » [MOCCOZET97] used free-form deformation to model hand deformation



Images courtesy of Sarah Frisken
Mitsubishi Electric Research Lab

Types of deformable models

- Physics-based models
  - Incorporates physical properties of model
    - » Pulling, cutting, tearing/breaking
  - More realistic deformations
    - » Model deforms intuitively according to applied force
  - Mass-springs, Finite-elements

Mass-Spring model

$$F = -k\Delta x$$

$$m\ddot{x} = -k\Delta x$$

$$m\ddot{x} = K\frac{(x_0 - x)}{|x_0|}$$

A simple mass-spring model

Stress, strain and Young’s modulus

Stress – force per unit area $\sigma = \frac{F}{A}$

Strain – change in length/initial length $\varepsilon = \frac{\Delta l}{l_0}$

Young’s modulus $K = \frac{\sigma}{\varepsilon}$

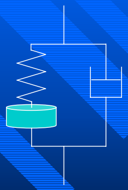
Mass-Spring model

$$m\ddot{x} = K\frac{(x_0 - x)}{|x_0|}$$

A mass-spring

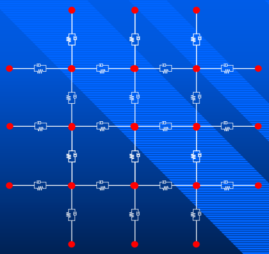
Mass-Spring model

$$m\ddot{x} = K\frac{(x_0 - x)}{|x_0|} - \gamma\dot{x}$$



Mass-spring with damper

Mass-Spring model



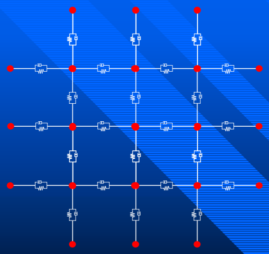
$$m\ddot{x}_i = -\gamma\dot{x}_i + \sum_{j\in N(i)} K_{i,j}\frac{(l^0_{i,j} - |x_i x_j|)x_i x_j}{l^0_{i,j}}$$

[DELINGETTE98]

$$m\ddot{x}_i = -\gamma\dot{x}_i + \sum_{j\in N(i)} K_{i,j}\frac{(l^0_{i,j} - |x_i x_j|)x_i x_j}{l^0_{i,j}} + f_i$$

$$M\ddot{x} + C\dot{x} + Kx = f$$

Mass-Spring model



$$m\ddot{x}_i = -\gamma\dot{x}_i + \sum_{j\in N(i)} K_{i,j}\frac{(l^0_{i,j} - |x_i x_j|)x_i x_j}{l^0_{i,j}} + f_i$$

$$M\ddot{x} = C\dot{x} + Kx + f$$

$$\ddot{x} = M^{-1}(C\dot{x} + Kx + f)$$

$$\dot{v} = M^{-1}(Cv + Kx + f)$$

$$(\dot{x} = v)$$

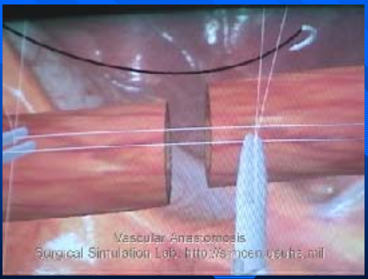
[GIBSON97]

Strengths

- Fast, requires relatively little computation
- Simple to understand
- "Realistic" for small deformations
  - Less accurate for large deformations

Mass-spring models

Mass-spring models



Mass-spring models

- Other examples of mass-spring models
  - Abdominal Trauma Surgery [BRO-NIELSEN98]
  - Laparoscopic Cholecystectomy Simulation [TENDICK00]
  - Modeling of musculature [NEDEL98]
  - Facial modeling [TERZOPOLUOS90]
    - » Multiple tissue layers

[BRO-NIELSEN98]

Mass-spring models

$$M\frac{d^2\underline{x}}{dt^2}+\Gamma\frac{d\underline{x}}{dt}+\underline{g}(t,\underline{x})=\underline{f}(t)$$

$\underline{g}$ Vector of inner forces

$\underline{f}$ Vector of external forces

Source: Forschungszentrum Karlsruhe

Weaknesses

- Need careful design of topology
  - e.g., modeling homogeneity
- Can be difficult to define spring parameters
  - What is a "realistic" amount of stiffness?
- Time step a function of spring stiffness [DELINGETTE98]

$$K_c \approx \frac{M}{n\pi^2(\Delta t)^2}$$

$$K_c = critical\ stiffness$$

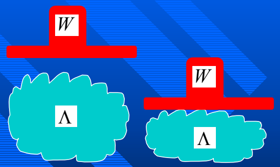
- The stiffer the spring(s), the smaller the time step.
- Can be a problem when modeling hard objects like bone.

Weaknesses

- Not an accurate physical model for tissue properties
  - Many tissues are not collections of springy tendons
- Becomes progressively less accurate for large deformations
- Need a framework that permits general physical principals to be represented

Deformation as energy

- "Shape" of tissue has potential energy
- Work done transfers energy to tissue
- Find the new tissue shape when energy is transferred to tissue.



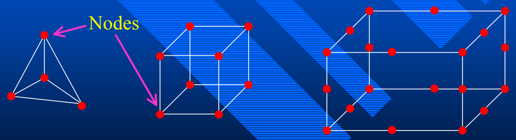
$$\Pi = \Lambda - W$$

Finite Element Models

- Relate potential energy to displacement of tissue from rest position.
  - Strain energy
- Relate work done as a function of tissue displacement.
- Compute tissue shape when Π is at minimum
  - Equilibrium
- Defined by
  - Shape elements, shape function, energy function

Finite Element Models

- Shape elements
  - Subdivide region of interest into discrete elements



Finite element models

- Shape function
  - Displacement of a point in the element is given as a function of displacement of the element's associated nodes
  - Typically polynomial
  - Equivalent to expressing strain of a point as a function of strain in nodes

$$\varepsilon(\vec{x}) = \sum_i f_i(\vec{x})\varepsilon_i$$

Strain energy

For linear elastic case, ε can be expressed in terms of displacement $\vec{u}$ by the following differential equations

$$\varepsilon_x = \frac{\partial \vec{u}}{\partial x} \qquad \varepsilon_{xy} = \frac{\partial \vec{u}}{\partial x} + \frac{\partial \vec{u}}{\partial y}$$

$$\varepsilon_y = \frac{\partial \vec{u}}{\partial y} \qquad \varepsilon_{xz} = \frac{\partial \vec{u}}{\partial x} + \frac{\partial \vec{u}}{\partial z}$$

$$\varepsilon_z = \frac{\partial \vec{u}}{\partial z} \qquad \varepsilon_{yz} = \frac{\partial \vec{u}}{\partial y} + \frac{\partial \vec{u}}{\partial z}$$

Finite element models

- Consider the energy function of one element
  - Strain energy of finite element
  - Work done to element
- At equilibrium, their sum is at a minimum
- Express this sum in terms of displacement

Strain energy

$$\Lambda = \frac{1}{2}\int_V \sigma^T \varepsilon \, \partial V = \frac{1}{2}\int_V \varepsilon^T D \varepsilon \, \partial V$$

[GIBSON97]

D = matrix of stress/strain components

Strain energy

$$\Lambda = \frac{1}{2}\int_V \sigma^T \varepsilon \, \partial V = \frac{1}{2}\int_V \varepsilon^T D \varepsilon \, \partial V$$

$$(\varepsilon = BU)$$

$$\Lambda = \frac{1}{2} U^T \Big(\int_V B^T D B \, \partial V\Big) U$$

[GIBSON97]

D = matrix of stress/strain components

U = composite vector of node displacements

B = matrix of differential equations relating position to strain

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Work done

$$W = \int_V u \cdot f \, \partial v$$

$$W = U^T F$$

U = composite vector of node displacements

F = composite vector of forces integrated over the object volume

= composite vector of equivalent forces acting at node points

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At equilibrium

Computing the minimum of

$$\Pi = \frac{1}{2} U^T \Big(\int_V B^T D B \, \partial V\Big) U + U^T F$$

yields

$$KU = F$$

K = stiffness matrix over the volume

- But!
  - Medical simulation is dynamic
  - Previous derivation is for static systems
- Extend to consider
  - Inertia
  - Damping
  - Similar to mesh-spring case

$$M\ddot{U} + C\dot{U} + KU = F$$

Strengths

- Can model complex soft tissue deformations more accurately
- Equations for Λ can be based on mechanical tissue models
- Capable of modeling non-linear tissue properties
  - – Resultant deformations are more realistic

Weaknesses

- Straightforward implementation is S-L-O-W
- Considerably slower than mesh-spring approach
  - – But see various speed-up methods
    - » Condensation
    - » Preprocessing
    - » Adaptive FEM
    - » Hybrid methods [COTIN00]

Condensation

[BRO-NIELSEN96]

- Idea
  - – Internal nodes not visible/do not interact directly with observer
    - » Not interesting
  - – Can we not compute their displacements?
- Recall for (one element) in static case

$$Ku = f$$

Condensation

- Rewrite linear system as a block matrix

$$\begin{bmatrix} K_{ss} & K_{si} \\ K_{is} & K_{ii} \end{bmatrix} \begin{bmatrix} u_s \\ u_i \end{bmatrix} = \begin{bmatrix} f_s \\ f_i \end{bmatrix}$$

- We want new expression involving only surface nodes

$$K_{ss}^* \vec{u}_s = \vec{f}_s^*$$

Condensation

- From block matrix

$$K_{ss}^* = K_{ss} - K_{si} K_{ii}^{-1} K_{is}$$

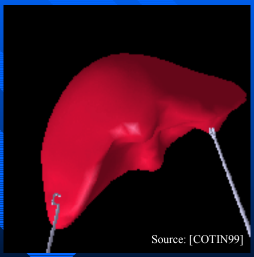
$$\vec{f}_s^* = \vec{f}_s - K_{si} K_{ii}^{-1} \vec{f}_i$$

- If there are no forces applied to internal nodes, then $\vec{f}_s^* = \vec{f}_s$

Preprocessing

[COTIN99]

- Preprocess “elementary deformations” for each free (movable) surface node.
- Apply combinations of linear deformations to achieve final deformation in real-time.



Source: [COTIN99]

Adaptive meshes

[WU01]

- Preprocess to get hierarchy of mesh resolutions and FEM matrices
- Adaptively refine based on threshold
  - Stress concentration
  - Displacement field
  - Optimized posterior error estimator
  - Stress gradient

Video courtesy of Frank Tendick

Current research issues

- Accuracy
  - Fidelity to the ways tissues actually behave
    - » Non-linear models
    - » Inhomogeneous models
  - How do tissues actually behave?
- Speed
  - Haptic rendering, visual realism
    - » Preprocessing
- Change in topology
  - Cutting and suturing

What is the purpose of your simulation?

- Develop motor skills
  - Laparoscopy, bronchoscopy, etc.
- Learning to do a procedure
  - Diagnostic peritoneal lavage, pericardiocentesis, central line placement, chest tube insertion
  - Practice for minimally invasive surgery
- Not every simulation needs absolutely realistic deformation modeling

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